or 6; but $s=6$ makes t impossible, while $s=5$ makes $t=4$, and we have $2st=40$ and $(s^2-t^2)=18$. So $2pq=80$ and $(p^2-q^2)=18$, as before. Take $m=50$, p may be 6 or 7, but 6 makes q impossible, while $p=7$ gives $q=1$, and $2pq=14$ and $p-q^2=48$. $\therefore$ $14^2+48^2=50^2$. But $50=5\times10$, and $5=4+1$; then $s=2$ and $t=1$, $2st=4$, and $s^2-t^2=3$, $2pq=40$, and $p^2-q^2=30$, and $30^2+40^2=50^2$. Solutions may be obtained from other factors, but they give one of these two results. If $m=65$, it may be shown in the same manner that there are 4 pairs of numbers answering the conditions.

II. Solution by R. J. ADCOCK, Larchland, Illinois.

Since $(x^2+y^2+z^2+u^2+v^2)^2=(x^2+y^2+z^2+u^2+v^2)^2+(2xv)^2+(2yv)^2+(2zv)^2+(2uv)^2$, is true for two or any greater number of letters. Then

$$\left[(2x+1)^2+(2y)^2\right]^2=\left[(2x+1)^2-(2y)^2\right]^2+\left[2(2x+1)2y\right]^2.$$

$$\text{And, } 1=\left[\frac{(2x+1)^2-(2y)^2}{(2x+1)^2+(2y)^2}\right]^2+\left[\frac{2(2x+1)2y}{(2x+1)^2+(2y)^2}\right]^2.$$

If n^2 be the given square number,

$$n^2=\left[\frac{(2x+1)^2-(2y)^2}{(2x+1)^2+(2y)^2}\times n\right]^2+\left[\frac{2(2x+1)2yn}{(2x+1)^2+(2y)^2}\right]^2 \text{are the}$$

two square parts into which n^2 is divided. Observing that x may have any value including 0, y any value not $=0$, and in order to avoid repetion of parts into which the square number is divided, the numbers for x and y must not make $2x+1$ and $2y$ have a common factor or be equal. If $x=0$, and $y=1$, the two square parts of n^2 are $\left(\frac{3n}{5}\right)^2$ and $\left(\frac{4n}{5}\right)^2$.

Also solved by *H. W. Draughon*, *G. B. M. Zerr*, *P. S. Berg*, *A. L. Foote*, *P. H Philbrick*, and *H. C. Whitaker*.

2. Proposed by J. M. COLAW, Principal of High School, Monterey, Virginia.

Find two numbers, such that the difference of their squares may be a cube, and the difference of their cubes a square.

I. Solution by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Let $6x^3$, $10x^3$ be the numbers.

Then $100x^6-36x^6=64x^6=$ a cube number, and $1000x^9-216x^9=784x^9=(28)^2x^9=$ a square number, when x is a perfect square. Let $x=1$, then 6 and 10 are the numbers. Other values can be obtained by substituting 4, 9, 16, etc. for x.

II Solution by C. W. M. BLACK, Department of Mathematics in Wilmington Conference Academy, Dover, Delaware.

(A). Represent the numbers by x, and $x+a$. The difference of the cubes will be (1), $a(3x^2+3ax+a^2)$, and the difference of the squares (2), $a(2x+a)$. For one solution the first will be a square if $a=3x^2+3ax+a^2$, and the second a cube if $a=(a+2x)^2$. Solving these equations, $a=1$ and $x=-1$ or 0, $a=0$ and $x=0$, making the required numbers -1 and 0, 0 and 1, or 0 and 0. Also, $a(2x+a)$ will be a cube if (3), $a^2=a+2x$. Combining (1) and (3), $(a-1)(3a^2+3a+4)=0$. The first factor gives $a=1$, whence $x=-1$ or 0, as before. The second factor gives an imaginary result. As these results are not satisfactory, we seek another method for finding other values, if there are any.

(B). It may easily be shown that if x and $x+a$ are positive intergers

and prime to each other, there is in general no solution. So, we have only to find how two numbers not prime to each other may be obtained to fulfill the requirements

(C). Let n and p be two positive integers prime to each other, then (4), $n^3-p^3=hr^2$, and (5), $n^2-p^2=ks^3$, where r, s, h, k, are positive integers. Let m be a factor such that if n and p are both multiplied by it, they will fulfill the required condition. Then, (6), $m^3n^3-m^3p^3=m^3hr^2=$ a square. (7), $m^2n^2-m^2p^2=m^2ks^3=$a cube. The first will be a square if $m=hc^2$ (8), and the second a cube if $m=kc^3$ (9). Combining(8)and (9),$c=\dfrac{h}{k}$and $m=\dfrac{h^3}{k^2}$ (10). Then by taking any two positive integers prime to each other, substituting, and finding values of h and k in equations (4) and (5), and multiplying each member by the value of m from (10),we have two numbers which fulfill the requirements. There is nothing in above proof to limit the method to positive integers prime to each other,but these may be taken as the basis of all positive numbers. When one of the numbers taken is negative,different results are derived. When both are negative, the value derived for m will change the numbers so that the difference of the cubes will not be negative. (Ex. 1) $5^3-3^3=98=2\times7^2$, $5^2-3^2=16=2\times2^3$, $h=2$, $k=2$, $m=2$. $\therefore$ numbers are 10 and 6. (Ex. 2). $2^3-1^3=7$, $2^2-1^2=3$, $h=7$, $k=3$, $m=\dfrac{7^3}{3^2}$. $\therefore$ numbers are $\dfrac{686}{9}$ and $\dfrac{343}{9}$. (Ex. 3).

$2^3-(-1)^3=9$, $2^2-(-1)^2=3$, $h=1$, $k=3$, $m=\dfrac{1}{9}$. $\therefore$ numbers are $\dfrac{2}{9}$ and$-\dfrac{1}{9}$. (Ex. 4). $1^3-0^3=1$, $1^2-0^2=1$, $h=1$, $k=1$, $m=1$. $\therefore$ numbers are 1 and 0,as derived before in (A).

Also solved by *H. W. Draughon*, *A. L. Foote*, *P. H. Philbrick*, and *G. B. M. Zerr*.

3. Proposed by O. S. KIBLER, Superintendent of Schools, West Middleburg, Logan County, Ohio.

It is required to find three whole numbers in an arithmetical progression, such that the sum of every two of them shall be a square.

Solution by A. L. FOOTE, 80 Broad St., New York City.

Let $\frac{1}{2}x^2-y$, $\frac{1}{2}x^2$, and $\frac{1}{2}x^2+y$ be the numbers. Then we must have x^2, x^2+y, and x^2-y each squares and since x^2 is a square, we require only to make x^2+y and x^2-y squares. Let $x^2+y=m^2$ and $x^2-y=n^2$. Let $y=2sx+s^2$ and we have $x^2+y=x^2+2sx+s^2=(x+s)^2$, and so we have but to find $x^2-2sx-s^2=n^2$. Now let $x^2-2sx-s^2=(x-m)^2=x^2-2mx+m^2$ and we have $x=\dfrac{m^2+s^2}{2m-2s}$ where m and s may be assumed at pleasure. Let $m=5$ and $s=4$, then $x=\dfrac{25+16}{10-8}=\dfrac{41}{2}$ and $\frac{1}{2}x^2=\dfrac{1681}{8}$, $y=2sx+s^2=164+16=180$, so we have $\dfrac{1681}{8}$, $\dfrac{241}{8}$, and $\dfrac{3121}{8}$. To render these integral multiply each by 16 and we have 482, 3362 and 6242 for the required numbers. The squares are $3844=62^2$, $6724=82^2$, and $9604=98^2$. The values of m must be so chosen that $\frac{1}{2}x^2-y$ will be positive.

J. H. Drummond finds 380, 8450 and 16514. Also solved by *P. S. Berg* and *H. W. Draughon*.